Multi-Branch Hidden Semi-Markov Modeling for RUL Prognosis

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SUMMARY & CONCLUSIONS

Deterioration modeling and remaining useful life (RUL) estimation of equipment are key enabling tasks for the implementation of a predictive maintenance (PM) policy, which plays nowadays an important role for maintaining engineering systems. Hidden Markov Models (HMM) have been used as an efficient tool for modeling the deterioration mechanisms as well as for estimating the RUL of monitored equipment. However, due to some assumptions not always justified in practice, the applications of HMM on real-life problems are still very limited. To tackle this issue and to relax some of these unrealistic assumptions, this paper proposes a multi-branch Hidden semi-Markov modeling (MB-HSMM) framework.

The proposed deterioration model comprises several different branches, each one being itself an HSMM. The proposed model offers thus the capacity to 1) explicitly model the sojourn time in the different states and 2) take into account multiple co-existing and competing deterioration modes, even within a single component. A diagnosis and RUL prognosis methodology based on the MB-HSMM model is also proposed. Thanks to its multiple branches property, the MB-HSMM model makes it possible not only to assess the current health status of the component but also to detect the actual deterioration mechanism. Based on the diagnostic results, the component RUL can then be calculated.

The performance of the proposed model and prognosis method is evaluated through a numerical study. A Fatigue Crack Growth (FCG) model based on the Paris-Erdogan law is used to simulate deterioration data of a bearing under different operation conditions. The results show that the proposed MB-HSMM gives a very promising performance in deterioration mode detection as well as in the RUL estimation, especially in the case where these deterioration modes exhibit very different dynamics.

1 MOTIVATION AND PROBLEM STATEMENT

Predictive Maintenance (PM) plays nowadays an important role in the maintenance of engineering systems since it has the ability to recommend in advance proper maintenance actions by based on the relevant information collected from condition monitoring [1]. Within a PM program, diagnosis and prognosis are two key enabling steps: diagnosis deals with fault detection, isolation and identification as well as health assessment of the monitored system; prognosis aims at predicting the evolution of the fault in the future and estimate the remaining time before a failure occurs. An accurate estimation of the RUL could provide ample time for maintenance engineers to schedule a repair or to acquire replacement components before they actually fail [2]. Due to the stochastic nature of the deterioration mechanisms, stochastic processes are well-suited for modeling the deterioration processes and have shown promising results within the PM framework [5, 6, 7, 8, 9, 13].

As doubly embedded stochastic processes, Hidden Markov Models (HMM) have been successfully applied in many domains, such as speech recognition, genes and deoxyribonucleic acid (DNA) analysis, etc. [4] thanks to their strong mathematical basis. In the PM context, HMM classify equipment conditions among several meaningful discrete states, such as “normal”, “minor defect”, “maintenance required” and “failure” and therefore can give easy-to-interpret results for maintenance implementation [5]. These models can also characterize the stochastic relationship between the features extracted from condition monitoring data and the actual health states of the equipment. For these reasons, HMMs in the recent year are being more and more investigated to be used as an efficient tool for deterioration modeling as well as for RUL estimation [3]. For example, Baruah et al. employed HMMs for carrying out both the diagnosis and the prognosis for metal cutting tools [6]. In [2], HMMs are used to build a health/degradation index representing the current system health status. This index is then fed into an adaptive prognostic scheme in order to estimate the RUL of a bearing. However, due to its inherent Markov property, the sojourn time in a state of the HMM model is geometrical or exponential distributed, which is not often true in practice. The semi-Markov based models could overcome this problem by allowing arbitrary probability distributions such as Gaussian or Gamma for the state sojourn time, [4]. From this point, several extensions have been adopted in the literature, such as Hidden semi-Markov models [7], segmented HMMs [8], segmented Hidden Semi-Markov Models [9], etc.

To the best of our knowledge, almost all of the Markov-based modeling frameworks for diagnosis and prognosis in the

literature deal with the mono-mode case, meaning that only one degradation mechanism is allowed to take place at one time. For example, the authors in [7] use different HSMMs to represent different health states of a pump but these states belong to only one deterioration mode. Or in [8], only one degradation mechanism of a milling machine is considered and modeled by one segmented HMM. In real cases, however, several degradation modes can co-exist in competition even within a single component. For instance, different internal elements of bearings can be subjected to degradation due to different operating conditions, leading to different rates in deterioration evolution of the bearings. To take into account such phenomena, we propose a semi-Markov based structure called multi-branch Hidden semi-Markov Model (MB-HSMM). A simulation study is carried out in order to evaluate the effectiveness of the proposed model with respect to a standard HSMM one.

The remainder of the paper is organized as follows. Section 2 is devoted to review some basic theories of HMM and HSMM as well as the extension to MB-HSMM model. Section 3 describes a framework based on the proposed model for diagnosis and prognosis. To illustrate the methodology, a comparative study is introduced in the section 4. Conclusion follows in Section 5.

2 THEORETICAL BACKGROUND

2.1 *Basic theory of Hidden Markov Model*

Hidden Markov Model consists of an underlying Markov chain which is not directly observable but can be revealed through observation [4]. Within the deterioration modeling framework, the hidden states represent the health states of equipment, while the observations can be either measurable raw data such as vibration signals or high-level features extracted from condition monitoring data. The relation between the hidden states and the observations is often represented by probabilistic models.

Mathematically, the complete specification of an HMM consists of the following elements [4]

- N, the number of states in the model
- Q, the number of distinct observation symbols per state
- A finite set of hidden states, i.e., $S = \{S_1, S_2, ..., S_N\}$
- A state transition probability distribution, i.e., $A = \{a_{ij}\}$, where $a_{ij} = P(q_{t+1} = S_j \mid q_t = S_i)$
- An observation symbol probability distribution, i.e., $B = \{b_j(k)\}$ where $b_j(k) = P(Y_k \mid q_t = S_j)$, $1 \le k \le Q$
- An initial state probability distribution, i.e., $p = \{p_i\}$ where $p_i = P\{q_t = S_i\}$, $1 \le i \le N$

A complete HMM is often represented by a compact notation $\lambda = (p, A, B)$ for convenience.

There exist several types of HMMs, such as ergodic or left-right (Bakis) HMMs [4]. Since the deterioration processes are often irreversible in practice, meaning that equipment cannot come back from a worse health state to a better one as time progresses, the left-right topology is chosen for the deterioration modeling purpose in this paper.

In the framework of machine condition monitoring, measures are usually continuous signals, such as the vibration amplitudes, motor current, etc. The observation sequences should therefore be modeled by continuous probability density functions (pdf) to obtain a better performance in the diagnosis and the prognosis [3]. For this reason, the Gaussian distribution will be adopted for the observations in this paper.

2.2 *Hidden semi-Markov Model (HSMM)*

In standard HMM models, due to the Markovian assumption the sojourn time in a state is either geometrical distributed in the discrete time case or exponential distributed in the continuous time case. As argued in [10], this can be a source of inaccurate duration modeling with the HMMs since most real-life systems do not exhibit this property. To overcome this problem, the underlying Markov process can be replaced with a semi-Markov one resulting in a more general model called Hidden semi-Markov Model (HSMM).

Unlike the standard HMM, the state sojourn time in HSMMs can follow any arbitrary distribution, such as Gamma or Gaussian, etc. Once entering a state, the process stays in the state for a period of time whose length is determined by the arbitrary distribution. At the end of this period, the process randomly moves to another state according to the state transition probability matrix A. Clearly, HMMs are a special class of HSMMs.

As in the previous section, the underlying semi-Markov process in the HSMM is assumed to be strictly left-right and the observations are Gaussian distributed. Furthermore, the sojourn time in a state is also modeled by continuous Gaussian distributions. Similarly to the standard HMM, an HSMM can be characterized by the following parameters: the initial state distribution p, the transition model A, the state duration distribution denoted by D, and the observation model B. Thus, an HSMM can be written as $\lambda = (p, A, D, B)$.

2.3 *Multi-branch Hidden semi-Markov model (MB-HSMM)*

As already stated, standard HSMMs are only suitable for modeling one deterioration mode at a time. To deal with the co-existence of multiple competing deterioration modes, the concept of multi-branch model has recently been introduced [15]. However, due to the Markovian assumption in the model, limitation in the sojourn time modeling still exists. In this section, we extend the model in [15] to a so called multi-branch HSMM for modeling multiple deterioration mechanisms of equipment. An example of a left-right multi-branch HSMM is illustrated in Figure 1.

The proposed model consists of several branches, each one representing one deterioration mode. Some important assumptions have been made in constructing the model. Firstly, it is assumed that once the degradation has started following a given path, the equipment follows this deterioration mode until it reaches the end of life, i.e. there is

no allowed switch between branches. The second assumption is that, the normal condition can last very long with no abnormal sign in the observations. The observations emitted in this state are not hence of interest in the framework of deterioration modeling. Furthermore, once the equipment has failed, it is stopped and no more measures can be obtained. From these points, the initial S_0 as well as the failed S_f states are considered as dummy states with no emission of observation in our model. Therefore, each branch can be seen as a single HSMM model and is assigned an *a priori* probability, Figure 1.

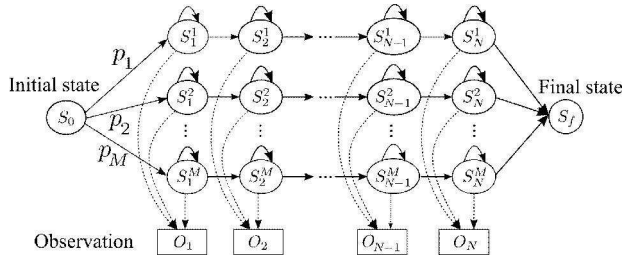

Figure 1 – A left-right multi-branch HSMM model.

From a training data set, these *a priori* probabilities can be estimated by:

$$p_k = P(\lambda_k) = K_k / K, \qquad \sum_{k=1}^{M} p_k = 1 \tag{2}$$

where λ_k denotes the constituent HSMM for branch k, K_k is the number of training data sequences corresponding to the mode k, K is the number of the total training data sequences and M is the total number of the branches.

3 MB-HSMM FOR DIAGNOSIS AND PROGNOSIS

In this section, a two-phase framework based on the MB-HSMM model is proposed for the diagnosis and the prognosis (c.f. Figure 2). It consists of two phases: off-line and on-line.

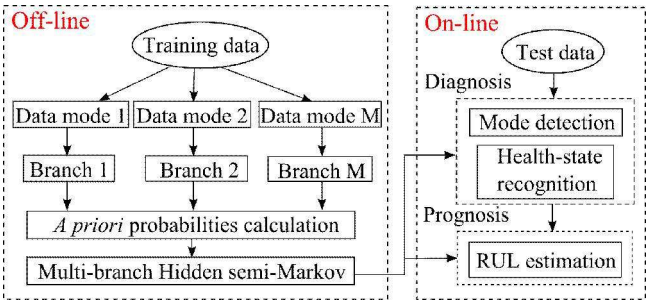

Figure 2 – MB-HSMM framework for diagnosis and prognosis

3.1 *Off-line phase*

This phase aims to learn an MB-HSMM model from the historical data. The idea is to train each branch individually and then combine them to obtain a final model. To do this, we firstly divide the training data set into M different groups in which each one corresponds to a deterioration mode. In this study, we suppose that the number of modes M is known beforehand thanks to the expert knowledge. Clustering techniques such as "kmeans" or "k-NN", etc. can hence be employed for the classification. Each data group is then used to train a constituent HSMM by using the Baum-Welch algorithm [4] in which the forward-backward (FB) procedure is the key process for parameters learning and likelihood estimation. We adopt the FB procedure proposed by Yu and Kobayashi in [14], which stands out to be the most efficient one.

After having trained the constituent branches, the *a priori* probabilities can be estimated by Equation (2) and a full MB-HSMM model is obtained.

3.2 *On-line phase*

In this phase, the trained model is used to assess the actual health state of the monitored equipment as well as to estimate its RUL. Due to the co-existence of multiple deterioration mechanisms, one must firstly identify the underlying one that is occurring. Indeed, wrong mode detection can lead to a wrong state assessment and inaccurate RUL estimation as well. In order to minimize the misclassification rate, the branch having the maximum *a posterior* probability given the data can be considered to correspond to the actual deterioration mechanism [11], that is:

$$\hat{k} = \arg\max_{k} P(\lambda_k \mid \mathbf{O}) \tag{3}$$

Using Bayes' theorem, these probabilities can be given by:

$$P(\lambda_k \mid \mathbf{O}) = \frac{P(\mathbf{O} \mid \lambda_k) P(\lambda_k)}{\sum_{k=1}^{M} P(\mathbf{O} \mid \lambda_k) P(\lambda_k)} \tag{4}$$

where $P(\mathbf{O} \mid \lambda_k)$ is the likelihood of the model λ_k given the data $\mathbf{O}$ and can be calculated through the forward-backward algorithm [14]. $P(\lambda_k)$ is calculated by Equation (2). After having identified the deterioration mode, the next diagnosis stage is to recognize the actual health state of the equipment. This can be carried out thanks to the Viterbi algorithm [4]. Given the detected mode, the most probable single state sequence (path) is the one that gives the maximum joint probability of the path and the observations:

$$Q^* = \arg\max_{Q_{\hat{k}}} P(\mathbf{O}, Q_{\hat{k}} \mid \lambda_{\hat{k}}) \tag{5}$$

where Q_k represents a possible path under the mode k. The actual health state of the equipment is considered as the final one of the path Q^*.

3.3 *RUL calculation*

Given the deterioration mode and the current health state estimated from the diagnostic stage, the RUL can be estimated in the same way as in a single HSMM case. However in the MB-HSMM framework, this estimation depends strongly on the mode detection results. Indeed, detecting the wrong mode may lead to a large bias in the RUL estimation. For example, if a crack is propagating with a high rate, the wrong detection of a low deterioration mode will lead to a quite high RUL value with respect to the actual one. This phenomenon often

occurs at the beginning of the defect propagation since observations are insufficient to show an obvious trend of the deterioration. In this study, the Bayesian Model Averaging (BMA) technique [12] is adopted to tackle the problem. The RUL distribution is calculated as an average of the posterior distributions under constituent modes, weighted by their posterior mode probabilities:

$$P(\mathrm{RUL}\,|\,\mathbf{O}) = \sum_{k=1}^{M} P(\mathrm{RUL}\,|\,\lambda_k, \mathbf{O}) P(\lambda_k\,|\,\mathbf{O}) \qquad (6)$$

where $P(\lambda_k\,|\,\mathbf{O})$ is given from Equation (4).

The procedure can now turn to the estimation of the RUL for each individual constituent HSMM model, which is straightforward. Assuming that equipment goes through degraded states $S_1, S_2, ..., S_N$ before reaching the failure state S_f and is in the state S_i at the current time t. Since the model is strictly left-right, the RUL can be calculated as the summation of two terms: the residual time of staying in the current state and the duration for the equipment to stay in the future health states before entering the failure one. Denote D_i and D_i^t the random variables representing respectively the total and the residual sojourn time in the state S_i for $i = 1, 2, \ldots N$, we have:

$$\mathrm{RUL}_i^t = D_i^t + \sum_{j=i+1}^{N} D_j \qquad (7)$$

Intuitively, the residual time is a conditional random variable $D_i^t = D_i - \bar{D}_i \,|\, D_i > \bar{D}_i$ where $\bar{D}_i$ is the time that the equipment has spent in the state S_i which can be calculated thanks to the Viterbi algorithm from the diagnostic stage. Given that D_i is Gaussian distributed, e.g. $D_i \sim \mathrm{N}(\mu_i, \sigma_i)$, it can be deduced that the residual time $D_i^t \,|\, D_i^t > 0$ follows a Normal distribution with mean $\mu_i - \bar{D}_i$ and standard deviation σ_i but truncated to the left of zero.

Besides that, due also to the Gaussian assumption, the sum element in Equation (7) is Normal distributed. Denote $Z = \sum_{j=i+1}^{N} D_j$, we have $Z \sim \mathrm{N}(\mu_z, \sigma_z)$ where $\mu_z = \sum_{j=i+1}^{N} \mu_j$ and $\sigma_z = \sqrt{\sum_{j=i+1}^{N} \sigma_j^2}$. The RUL becomes the summation of a truncated Normal distribution D_i^t and a Normal distributed variable Z. The cumulative distribution function of the RUL is therefore given by [16]:

$$F_{RUL}(x) = \frac{1}{1 - \Phi\left(\frac{a}{\sigma}\right)} \int_{-\infty}^{x-a} \left[\Phi\left(\frac{(x-u)}{\sigma}\right) - \Phi\left(\frac{a}{\sigma}\right) \right] \phi(u)\, du \qquad (8)$$

where $a = -\frac{\mu_i - \bar{D}_i}{\sigma_z}$, $\sigma = \frac{\sigma_i}{\sigma_z}$ and $\phi(\xi) = \frac{1}{\sqrt{2\pi}} e^{-\xi^2/2}$ is the probability density function of the standard normal distribution and $\Phi(\cdot)$ is its cumulative distribution function.

After evaluating all the individual branches, the RUL estimation under the MB-HSMM can be accomplished by Equation (6).

4 NUMERICAL EXAMPLES

To evaluate the proposed MB-HSMM-based diagnosis and prognosis performance, the data representing several different deterioration paths have been generated by the Fatigue Crack Growth (FCG) model. The FCG model is chosen since it has been widely used in the literature to describe the propagation process of a crack within a bearing [13]. The bearing is supposed to operate under different operation conditions so that the crack can propagate at different rates, which represent different deterioration modes.

4.1 *Simulated fatigue crack growth data*

In this section, we use the discretized and randomized FCG model in [13]:

$$x_{t_i} = x_{t_{i-1}} + e^{w_{t_i}} C \left(\beta_b e^{\gamma_e} \sqrt{x_{t_{i-1}}} \right)^n \Delta t \qquad (9)$$

where x_{t_i} represents crack depth at time t_i, C and n are constants depending on the material property, the exponential element $e^{w_{t_i}}$ helps to take into account the stochastic aspect of the crack propagation, β_b is the base stress level of component, the values $\gamma_e \geq 0$, $e = 1, 2, \ldots, M$ determine the level of extra stress linking with the state e of the environment.

Since it is often difficult to measure directly and accurately the crack depth in practice, an observation model has been adopted to represent the relationship between the actual values and the measure ones:

$$y_{t_i} = x_{t_i} + \xi_{t_i} \qquad (10)$$

where y_{t_i} is the measurement at time t_i and $\xi_{t_i} \sim N(0, \sigma_\xi^2)$ is the measurement error.

4.2 *Diagnosis and prognosis results*

In this section, the bearing is supposed to operate under two different environments and therefore cracks propagate possibly at two different rates. A two-branch HSMM model (cf. figure 1) is implemented to represent these two modes.

The following parameters of the FCG model are chosen: $C = 0.005$, $n = 1.3$, $\beta_b = 1$, $\sigma_w = 1.7$, $\Delta t = 1$, $\sigma_\xi^2 = 10$, $\gamma_e = [0 \;\; 0.75]^T$, $L = 100$, $p_1 = p_2 = 0.5$ where L represents the critical level beyond which the bearing is supposed to fail and the superscript T denotes the transpose of a vector. For training purpose, 100 data paths are generated (Figure 3).

Based on the times to failure information, this data set can be separated in two different "clusters" thanks to the k-means algorithm. The *a priori* probabilities for the branches are then

calculated (c.f. Equation (2)) and we obtain: $\hat{p}_1 = 0.45$ and $\hat{p}_2 = 0.55$.

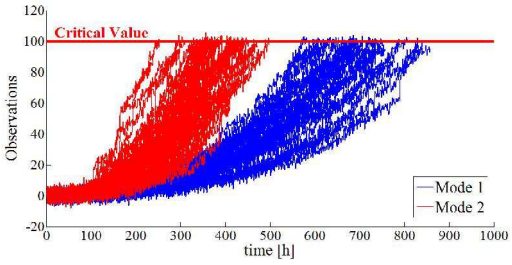

Figure 3 – Two-mode training data

To determine the number of states for each branch, the Bayesian Information Criterion (BIC) can be adopted as detailed in [15]. The candidate model having the minimum value of BIC will be chosen. In this study, the value $N = 8$ is obtained for every constituent model.

After having determined the number of states, each model is trained thanks to the Baum-Welch algorithm. An MB-HSMM is obtained and the offline phase is terminated.

We now move to the on-line phase. One data curve serving as the test data is also generated by the FCG model. In this study, the diagnosis and the prognosis procedures are repeated after each 50h as time progresses. The mean values of the estimated RUL associated with the 95% confidence interval are represented in Figure 4.

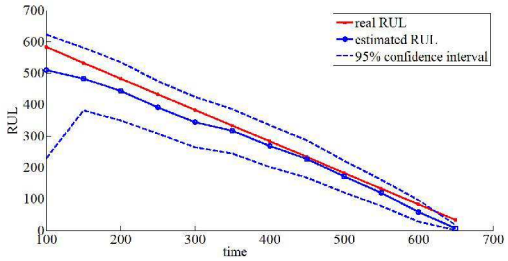

Figure 4 – RUL estimation at different time instances.

It can be seen that at early time instances, the lack of information due to the limited observations results in the bias and large variance in the RUL estimation. Even though, the real RUL values always lie within the 95% confidence interval. As the time passes, the width of the confidence interval significantly decreases and the estimated RUL values converge to the real ones. This demonstrates the accuracy of the proposed RUL estimation method.

4.3 *Multi-branch vs average HSMM*

An interesting question naturally rises in evaluating the advantage of the proposed approach: between the MB-HSMM and an "average" HSMM, which one does give the better RUL prediction results given the same data set. To answer such question, a comparative study is conducted in this section with 100 test deterioration paths. The prognostic performance is evaluated by the root mean squared error (RMSE) metric, where the errors are the differences between the mean estimated RULs and the actual values.

For notation convenience, we denote the average HSMM by AVG-HSMM. The training data set in the previous section is not grouped and is used to train only one AVG-HSMM. The trained model is then used in the on-line phase to estimate the RUL. The test data is the same as in the previous section. At the time $t = 100h$, we obtain $RMSE_{AVG} = 106.74$ and $RMSE_{MB} = 64.72$ for the AVG-HSMM and the MB-HSMM models respectively. It can be concluded that the proposed model give a better performance in RUL estimation than the average model in this case.

Another question raises naturally: does the above conclusion still hold in case that the two propagation rates are similar, or in case that they are quite different? To answer this question, it is necessary to define a "distance" between two deterioration modes. In this study, the mode "distance" is defined by the difference between the propagation rates in the FCG model. From Equation (9), it can be seen that the propagation rate of a crack is directly proportional to γ_e: the greater the value of γ_e is, the more quickly the crack propagates. Therefore, by fixing γ_1 and varying γ_2, the mode distance can be changed.

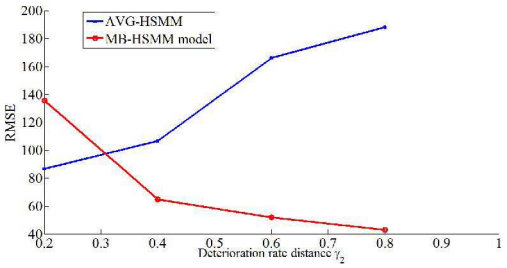

Figure 5 –RMSE at different mode distances.

Figure 5 shows that at the beginning when the two deterioration modes are similar, the multi-branch model gives a less accurate result than the one obtained with the average model. In this case, the simple average model is well-adapted to the data. Conversely, the multi-branch model may appear overly complex, leading to over fitting and poor predictive performances. However, once the mode distance increases, the MB-HSMM becomes more and more relevant and its RMSE decreases while it increases with AVG-HSMM. Hence, it can be concluded that the larger the distance between deterioration modes the better the performance in RUL estimation given by the proposed MB-HSMM approach, as compared to an "average" HSMM.

5 *CONCLUSION*

The present paper proposes a novel model called multi-branch Hidden semi-Markov Model (MB-HSMM) to deal with the deterioration modeling problem in case that there are several competing modes existing in the deterioration mechanisms of an equipment. A diagnostic and prognostic framework based on the proposed model is also given. The actual health state recognition and the RUL estimation can be implemented in the same way as in the traditional HSMM approach. However, by using different HSMM models for

different modes, together with the *a priori* probabilities of each model, the MB-HSMM shows a very promising result in dealing with the multi-mode deterioration mechanism.

Future research will be focused on the extension of the MB-HSMM to allow the switching between the states of the different branches. The proposed model and the extension should also be validated on the data of real systems.

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